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On thermal boundary layer on a power-law stretched surface with suction or injection

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Similarity solutions of the laminar boundary-layer equations describing heat and flow in a quiescent fluid driven by a stretched surface subject to suction or injection are obtained herein. The surface is moving with a power-law velocity distribution, and its temperature has a power-law variation. The effect of various governing parameters, such as Prandtl number Pr, temperature exponent n, velocity exponent m, and the injection parameter d, which determine the temperature profiles and heat transfer coefficient are studied. Three boundary conditions of uniform temperature, variable temperature, and uniform heat flux at the surface have been investigated. The effect of decreasing d is found to be significant, particularly for high Prandtl numbers.

Keywords: Stretched surface; power law; boundary layer

Introduction

A continuously moving surface through an otherwise quiescent medium has many applications in manufacturing processes. Such processes are hot rolling, wire drawing, metal extrusion, crystal growing, continuous casting, glass fiber production, and paper production (see Altan et al. 1979; Fisher 1976; Tadmor and Klein 1970). The study of heat transfer and the flow field is necessary for determining the quality of the final products of these processes as explained by Karwe and Jaluria (1988, 1991). This motivates our study to see the effect of suction or injection on the heat and flow boundary layer on a continuously stretched surface.

The resulting flow on a stretched surface extruded from a slit may be modeled as a boundary layer developing away from a slit. Sakiadis (1961a, b) was the first to develop a numerical solution for the flow field of a stretched surface using a similarity transformation. Tsou et al. (1967) reported the flow and heat transfer developed by a continuously moving surface both analytically and experimentally. He confirmed that the flow field obtained by Sakiadis (1961a,b) is broadly realized experimentally.

Many authors have attacked the problem for a plate moving with a linear velocity and for various temperature boundary conditions (Crane 1970; Grubka and Bobba 1985; Soundalgekar and Murty 1980; Vleggar 1977). Recently, Ali (1994) has reported flow and heat transfer characteristics on a stretched surface subject to a power-law velocity and temperature distributions for three different boundary conditions. Furthermore, the flow field of a stretching wall with a power-law velocity variation was discussed by Banks (1983) and recently by Ali (1996), who extended Banks's work for the stretched surface to be porous for different values of injection parameter.

Suction or injection of a stretched surface was introduced by Erickson et al. (1966) and Fox et al. (1968) for uniform surface velocity and temperature. Gupta and Gupta (1977) extended Erickson's work, in which the surface was moved with a linear speed for various values of parameters. Furthermore, linearly stretching surface subject to suction or injection was studied by Chen and Char (1988) for uniform wall temperature and heat flux, these authors also discussed the impermeable case with variable wall heat flux for different parameters.

In actual manufacturing process, the stretched surface speed and temperature play very important role in the cooling process (Karwe and Jaluria 1991). Furthermore, during the manufacture of plastic and rubber sheets it is often necessary to blow a gaseous medium through the not-yet-solidified material (Laksmisha et al. 1988). Although the boundary-layer-type solution is not true very close to the slit region, it has been used as a reasonable solution to model such processes as in the references cited earlier. Therefore, the present study focuses on using any combination of speed and temperature boundary conditions by employing the most general power-law velocity and temperature distributions with various injection parameters to model these processes as a laminar boundary-layer flow and heat over a continuous stretched surface.

Mathematical formulation

The motion of a laminar thermal active incompressible viscous fluid with constant properties describing zero pressure gradient boundary-layer flow on a stretched surface with suction or injection may be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

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Received 12 May 1994; accepted 26 November 1994

Int. J. Heat and Fluid Flow 16: 280–290, 1995

655 Avenue of the Americas, New York, NY 10010

0142-727X/95/$10.00
0142-727X(95)00001-7

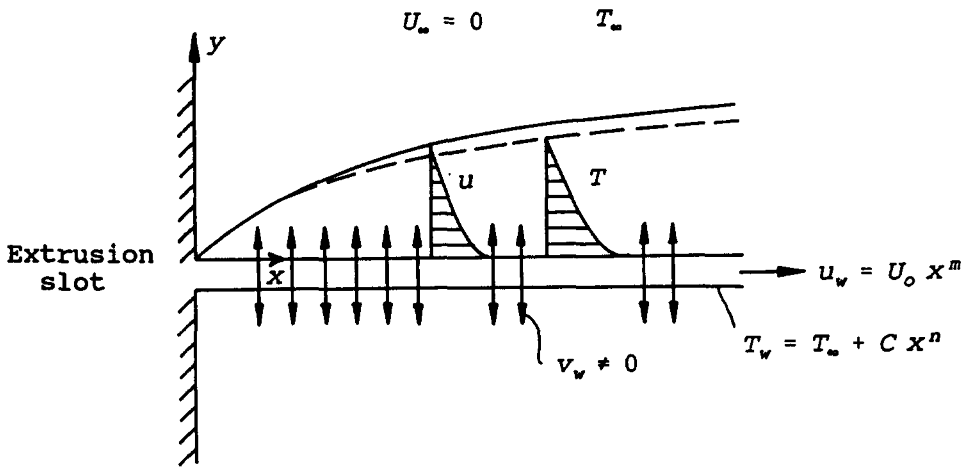


Figure 1 Schematic of flow induced by a stretched surface

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

subject to the following boundary conditions:

$$u = U_o x^m, \quad v = v_w(x) \quad @y = 0 \tag{4}$$

$$T - T_\infty = Cx^n \quad @y = 0 \tag{5}$$

$$u \to 0, \quad T \to T_\infty \quad @y \to \infty \tag{6}$$

It should be noted that positive and negative m indicate that the surface is accelerated or decelerated from the extruded slit respectively.

The x coordinate is measured along the moving surface from the point where the surface originates, and the y coordinate is measured normal to it, as shown schematically in Figure 1. Positive and negative v imply injection and suction, respectively. A similarity solution arises when

$$u = U_0 x^m f'(\eta), \quad T - T_\infty = Cx^n\Theta(\eta) \tag{7}$$

$$\eta = y\sqrt{\frac{m+1}{2}}\sqrt{\frac{U_0 x^m}{vx}} \tag{8}$$

$$v = -\sqrt{\frac{2vU_0}{m+1}}\, x^{\frac{m-1}{2}}\left(\frac{m+1}{2} f + \frac{m-1}{2} f'\eta\right) \tag{9}$$

where f' and Θ are the dimensionless velocity and temperature, respectively, and η is the similarity variable. Substitution in the governing equations gives rise to the following boundary-value problems

$$f''' + ff'' - \frac{2m}{m+1} f'^2 = 0 \tag{10}$$

$$\Theta'' + \Pr\left[f\Theta' - \frac{2m}{m+1} f'\Theta\right] = 0 \tag{11}$$

$$f'(0) = 1, \quad f(0) = -d\sqrt{\frac{2}{m+1}} \tag{12}$$

$$f'(\infty) \to 0, \quad \Theta(\infty) \to 0 \tag{13}$$

Notation

C	dimensional constant [(degree)·(length)$^{-n}$]
d	dimensionless injection parameter [$v_w(x^{1-m}/U_o v)^{1/2}$]
f	dimensionless stream function
k	thermal conductivity
m	velocity exponent parameter
n	temperature exponent parameter
Nu	Nusselt number (= hx/k)
Pr	Prandtl number (= v/α)
Re	Reynolds number [= $(U_o x^m)x/v$]
T	temperature
u	velocity component in x-direction
U_o	dimensional constant [(length/time)·(length)$^{-m}$]
v	velocity component in y-direction
x	coordinate in direction of surface motion
y	coordinate in direction normal to surface motion

Greek

α	thermal diffusivity
η	dimensionless similarity variable [= $y((m+1)/2)^{1/2}(U_o x^{m-1}/v)^{1/2}$]
Θ	dimensionless temperature [$=(T - T_\infty)/(T_w - T_\infty)$]; [$= k(T - T_\infty)\mathrm{Re}^{1/2}/xq_w(2/(1+m))^{1/2}$]
v	kinematic viscosity

Subscripts

w	condition at the surface
∞	condition at ambient medium

Superscripts

$'$	differentiation with respect to η
$-$	average quantity

In the equations above, primes denote order of differentiation with respect to η, Pr is the Prandtl number, and d is the dimensionless injection speed used to control the strength and direction of the normal flow defined by $v_w\ (x^{1-m}/U_o v)^{1/2}$ to permit a similarity solution. This solution is valid as long as the injection parameter d is such that the velocity profiles $f'(\eta)$ are beyond the separation limit where the boundary-layer assumptions are no longer valid. Three thermal boundary conditions of uniform temperature ($n = 0$), uniform heat flux, and variable temperature at the surface are considered, respectively

$$\Theta(0) = 1, \quad n = 0, \quad C = T_w - T_\infty \tag{14}$$

$$\Theta'(0) = -1, \quad n = \frac{1-m}{2}, \quad C = \frac{q_w}{k}\sqrt{\frac{2}{1+m}}\sqrt{\frac{v}{U_0}} \tag{15}$$

$$\Theta(0) = 1, \quad n \neq 0, \quad C = \frac{T_w - T_\infty}{x^n} \tag{16}$$

It should be noted that $\Theta(\eta)$ is defined as follows:

$$\Theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \Theta(\eta) = \frac{(T - T_\infty)k\sqrt{\mathrm{Re}}}{xq_w\sqrt{\dfrac{2}{m+1}}} \tag{17}$$

for variable or uniform surface temperature and for uniform surface heat flux, respectively, where $\mathrm{Re} = (U_o x^m)\ x/v$ is the Reynolds number. The local heat transfer coefficient h can be expressed in dimensionless form of Nusselt number for uniform surface heat flux and variable or uniform surface temperature, respectively.

$$\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}} = \frac{\sqrt{\dfrac{m+1}{2}}}{\Theta(0)}, \quad \frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}} = -\sqrt{\frac{m+1}{2}}\,\Theta'(0) \tag{18}$$

Furthermore, the average heat transfer coefficient $\bar{h}$ is related to the local coefficient by

$$\bar{h} = \frac{2h}{2n + m + 1} \tag{19}$$

It should be noted that for uniform surface heat flux $\bar{h} = h$, and the average Nusselt number follows from Equation 19 for variable surface temperature

$$\overline{\mathrm{Nu}} = -\frac{\sqrt{2(m+1)}\sqrt{\mathrm{Re}}\,\Theta'(0)}{(2n + m + 1)} \tag{20}$$

and for uniform surface temperature where $n = 0$, $\overline{\mathrm{Nu}}$ is given by the following:

$$\overline{\mathrm{Nu}} = -\sqrt{\frac{2}{m+1}}\sqrt{\mathrm{Re}}\,\Theta'(0) \tag{21}$$

Numerical method

The transformed momentum Equation 10 and the general energy Equation 11 subject to the boundary conditions 12–13 and any one of Equations 14, 15 or 16 were integrated numerically by the well-known fourth-order Runge–Kutta–Merson method. The half interval method was used to search for $f''(0)$ until $f'(\eta)$ decays exponentially to zero, the Gaussian elimination method was used to solve the energy equation, and the algorithm was modified following Chow (1983). The solution provided f, f', f'', and Θ, and the forward finite difference equation was employed to obtain $\Theta'(0)$ for variable and uniform temperature boundary conditions.

The numerical results were found to depend upon η_∞ and the step size. A step size of $\Delta\eta = 0.015$ and 0.005 gave sufficient accuracy for Prandtl number of 0.72 to 10, respectively. The value of η_∞ was chosen as large as possible between 4 to 13, depending upon the Prandtl number and the injection parameter, without causing numerical oscillations in the values of f', f'', and Θ. The computation was carried out on an IBM compatible 486 PC. Comparison is made with the available published data in Table 1 in terms of Nusselt number and in Table 2 in terms of temperature gradient at the wall. The fact that these results show a close agreement is an encouragement for further study of the effects of other various parameters on the stretched surface. On the other hand, a comparison was made with Gupta and Gupta (1977) for Prandtl number not equal one, however significant differences in $\Theta'(0)$ were found, which were also observed by Grubka and Bobba (1985).

Table 1 Comparison of $\mathrm{Nu/Re}^{1/2}$ for $m = 0$, $n = 0$, $d = 0$, and for various Prandtl numbers to previously published data

Pr	Jacobi (1993)	Soundalgekar and Murty (1980)	Chen and Strobel (1980)	Tsou et al. (1967)	Present results
0.7	0.3492	0.3508	0.34924	0.3492	0.3476
1.0	0.4438	—	—	0.4438	0.4416
10.0	1.6790	1.6808	—	1.6804	1.6713

Table 2 Comparison of $\Theta'(0)$ for $m = 1$, $n = 0$, and for various Prandtl numbers to previously published data

d	Pr	Grubka and Bobba (1985)	Lakshmisha et al. (1988)	Gupta and Gupta (1977)	Present results
0	0.7	—	0.45446	—	−0.45255
0	1.0	−0.5820	—	−0.5820	−0.59988
0	10.0	−2.3080	—	—	−2.29589
1.067	1.0	—	—	−0.1105	−0.10996

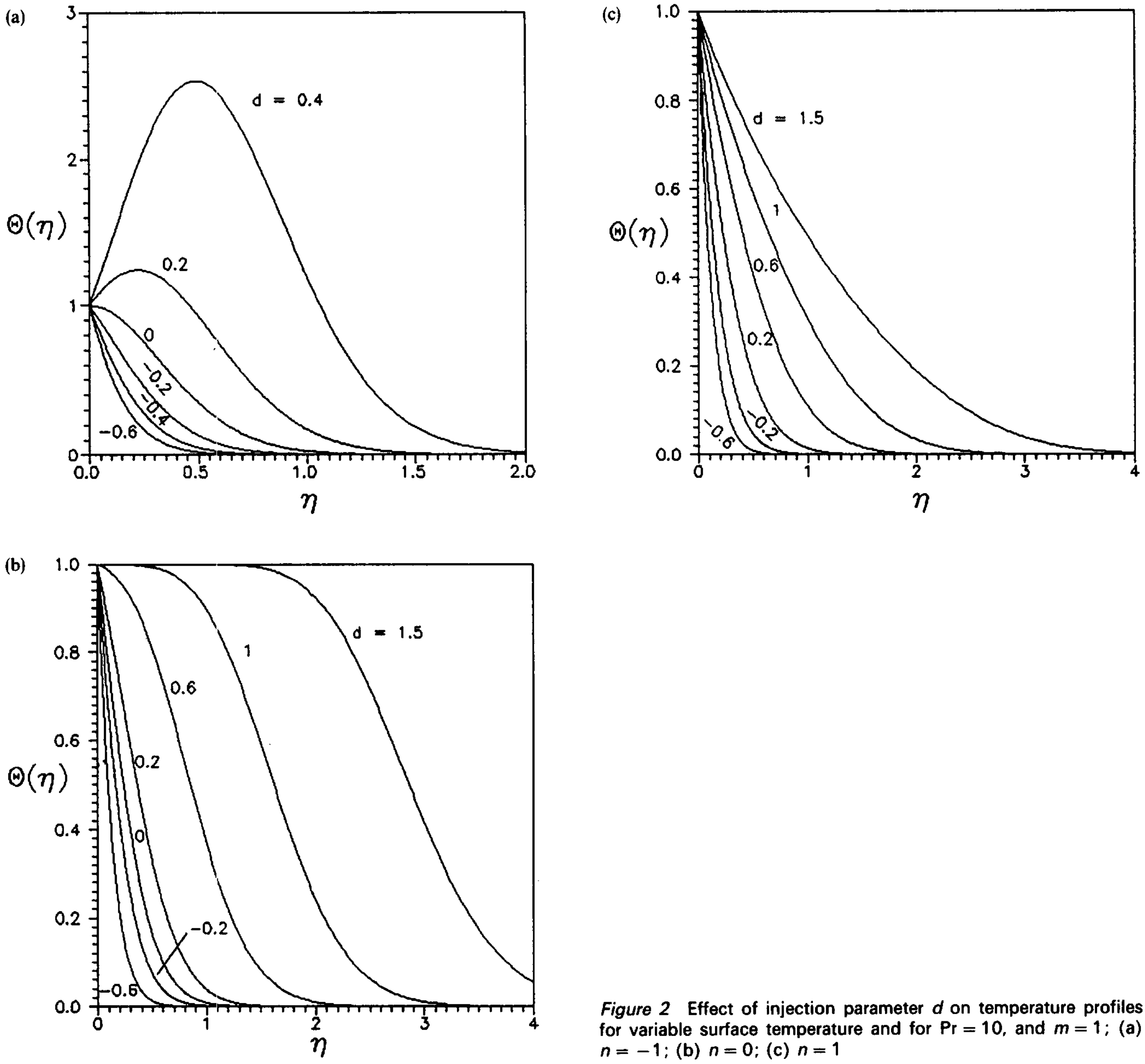


Figure 2 Effect of injection parameter d on temperature profiles for variable surface temperature and for $Pr = 10$, and $m = 1$; (a) $n = -1$; (b) $n = 0$; (c) $n = 1$

Results and discussion

Results for the dimensionless temperature profiles and Nusselt numbers are obtained for various values of Prandtl numbers 0.72, 1, 3, and 10 and for different values of d, m, and n. Three boundary conditions of uniform temperature, variable temperature, and uniform heat flux at the surface are considered.

Figure 2 shows samples of the dimensionless temperature profiles Θ for $Pr = 10$ and $m = 1$ as a function of the similarity variable η for various values of d. In Figure 2a, $n = -1$, and heat is transferred to the moving surface for the injection case ($d > 0$) and from the surface for suction ($d < 0$). As might be expected, suction thins the thermal boundary layer; whereas, injection thickens it. Figures 2b and c are for uniform surface temperature ($n = 0$) and for linear temperature distribution ($n = 1$), respectively. In these figures, the thinning of the boundary layer with blowing is evident for increasing the surface temperature from a uniform to a linear relation, and it can also be seen that all the heat was transferred from the surface to the medium. It should be noted that in Figure 2b, the boundary-layer assumptions do not permit a solution of the boundary-layer equation for large d because Θ will approach a constant value of 1, and the boundary layer is almost literally blown off the surface, similar to that of stationary plate with injection (Burmeister 1983; Kays and Crawford 1987).

The same trend can be seen for the temperature profiles for $Pr = 0.72$ in Figure 3a–c, where the thermal boundary-layer thickness is thicker than those for $Pr = 10$, as expected, and their gradient is less steep near the edge of the surface.

The local heat transfer coefficient $Nu/Re^{1/2}$ depends upon the blowing parameter d, as shown in Figure 4a–c for $n = -1$ and various Prandtl numbers of 0.72, 1, 3, and 10. In Figure 4a, m is -0.2, and a substantial influence on $Nu/Re^{1/2}$ is exerted by the blowing parameter d and Pr. Negative values of $Nu/Re^{1/2}$ indicate that heat flows into the surface despite the surface temperature's continual excess over the free-stream temperature. This physical mechanism could be explained as a

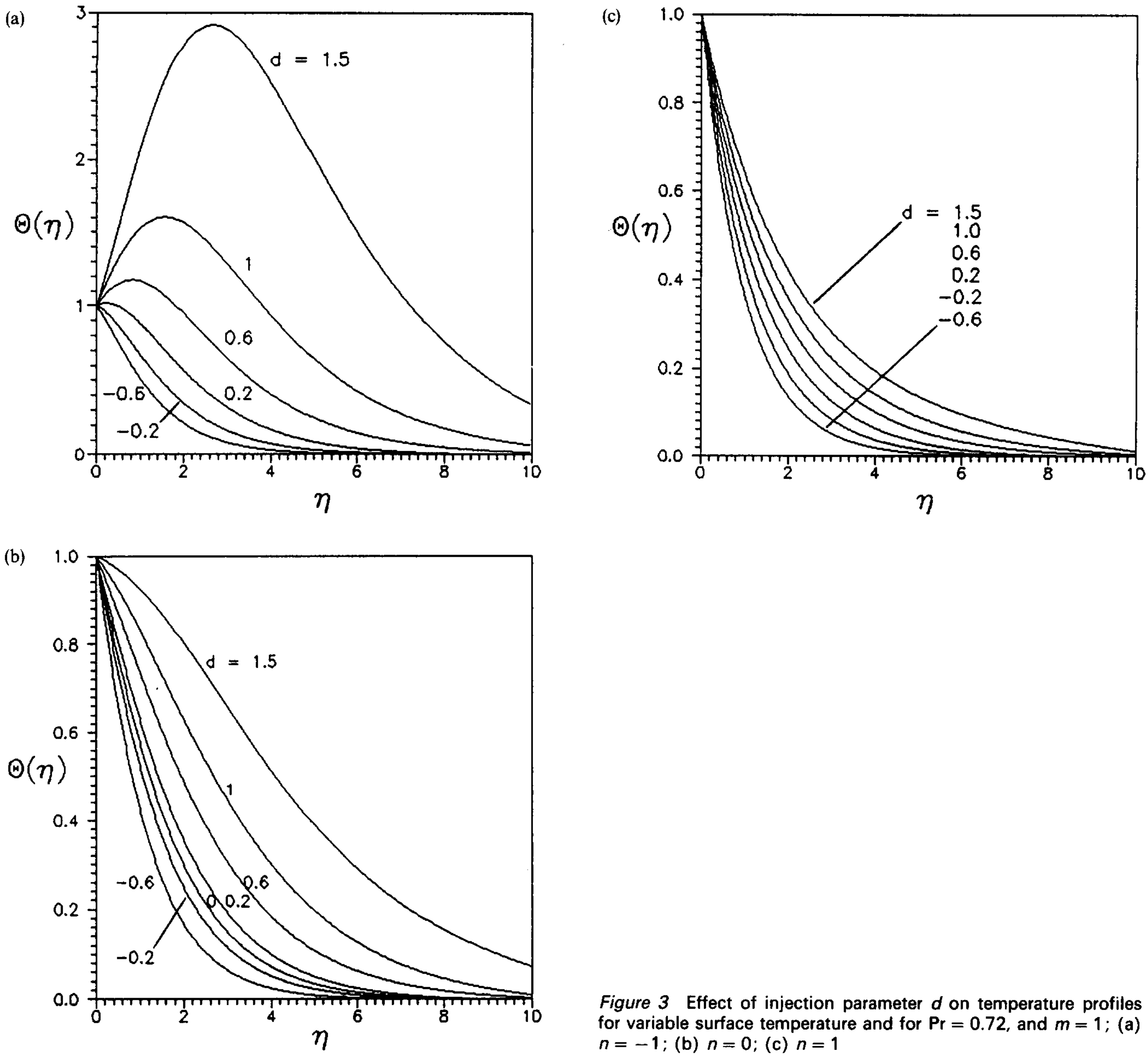


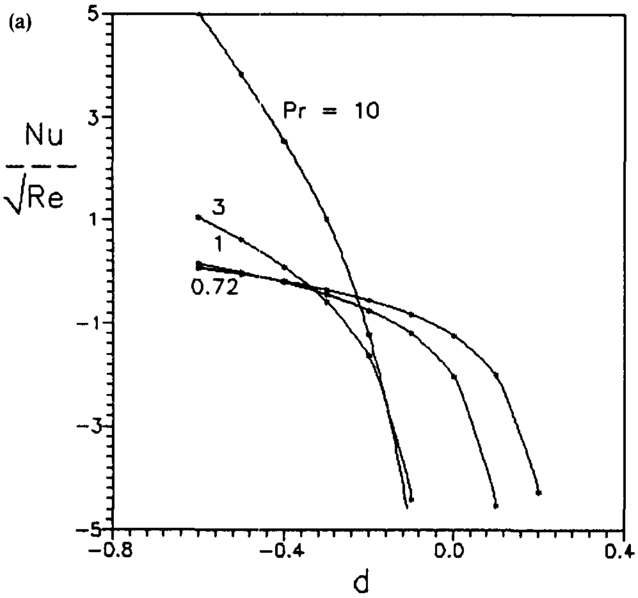
Figure 3 Effect of injection parameter *d* on temperature profiles for variable surface temperature and for Pr = 0.72, and $m = 1$; (a) $n = -1$; (b) $n = 0$; (c) $n = 1$

fluid particle heated to nearly the surface temperature being convected downstream to a place at which the surface temperature is lower. Then heat flows into the surface and results in negative heat transfer coefficients, which means only that $\Theta'(0)$ is no longer proportional to $(t_w - t_\infty)$ (see Burmeister 1983). However, positive values of $Nu/Re^{1/2}$ show that heat is transferred from the surface to the medium results in positive heat transfer coefficients. Figure 4b–c present slight enhancement of $Nu/Re^{1/2}$ for changing m from 1 to 5, respectively, for the same range of Pr as a function of the blowing parameter d. Comparing Figure 4a to that of 4c, for the same range of d, it is clear that the accelerated stretched surface ($m = 5$) improves the convection of the heated fluid particle away from the surface downstream where the surface has a temperature lower than that upstream. Thus, heat always flows from the surface to the medium enhances the heat transfer coefficient; however, the opposite is true for a decelerated surface ($m = -0.2$), where a negative heat transfer is developed depending upon Prandtl number (see Table 3 for $n = -1$ and $m = -0.2$, and 5).

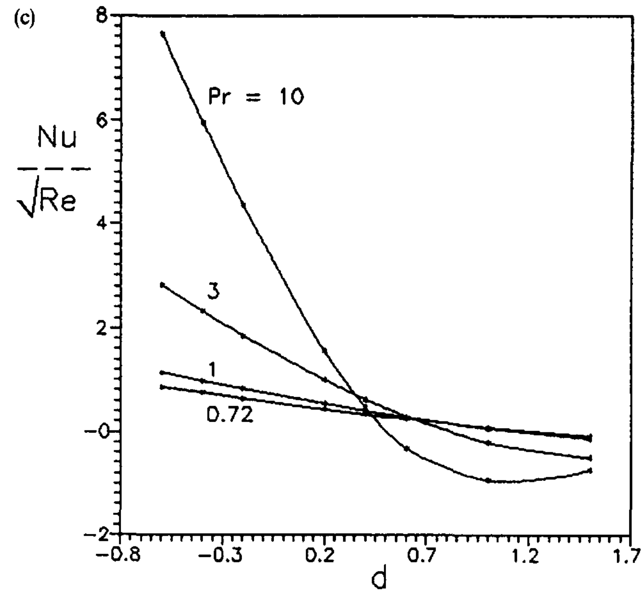
Figure 5a–c shows the same trend for $n = 1$ and for the same values of m as a function of the blowing parameter d, which indicates that increasing the surface temperature affects the direction of heat from the surface to the medium, where $Nu/Re^{1/2}$ is positive for all d investigated. Furthermore, in Figures 4 and 5 it is clear that suction ($d < 0$) enhances the heat transfer coefficient much better than blowing ($d > 0$), and the thickness of the thermal boundary layer is reduced, just as was previously found to be true for Pr. Thus, suction can be used as a means for cooling the surface much faster than blowing. Numerical values of $Nu/Re^{1/2}$ corresponding to variable surface temperature have been listed in Table 3 for various values of m, d, n, and Pr, physically unrealistic values are presented as dashed lines.

A sample of the temperature profiles for the uniform surface heat flux is illustrated in Figure 6 for Pr = 0.72, various values

(a)



(c)



(b)

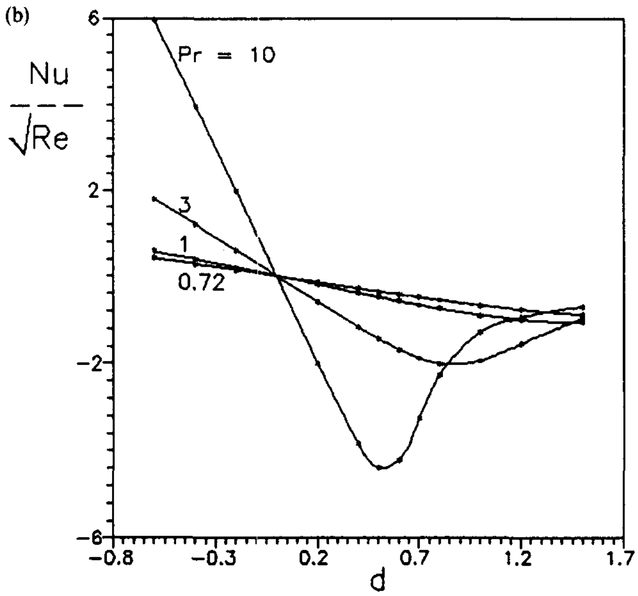


Figure 4 Variation of $Nu/Re^{1/2}$ as a function of the injection parameter d for variable surface temperature, $n = -1$, and for various values of Prandtl number; (a) $m = -0.2$; (b) $m = 1$; (c) $m = 5$

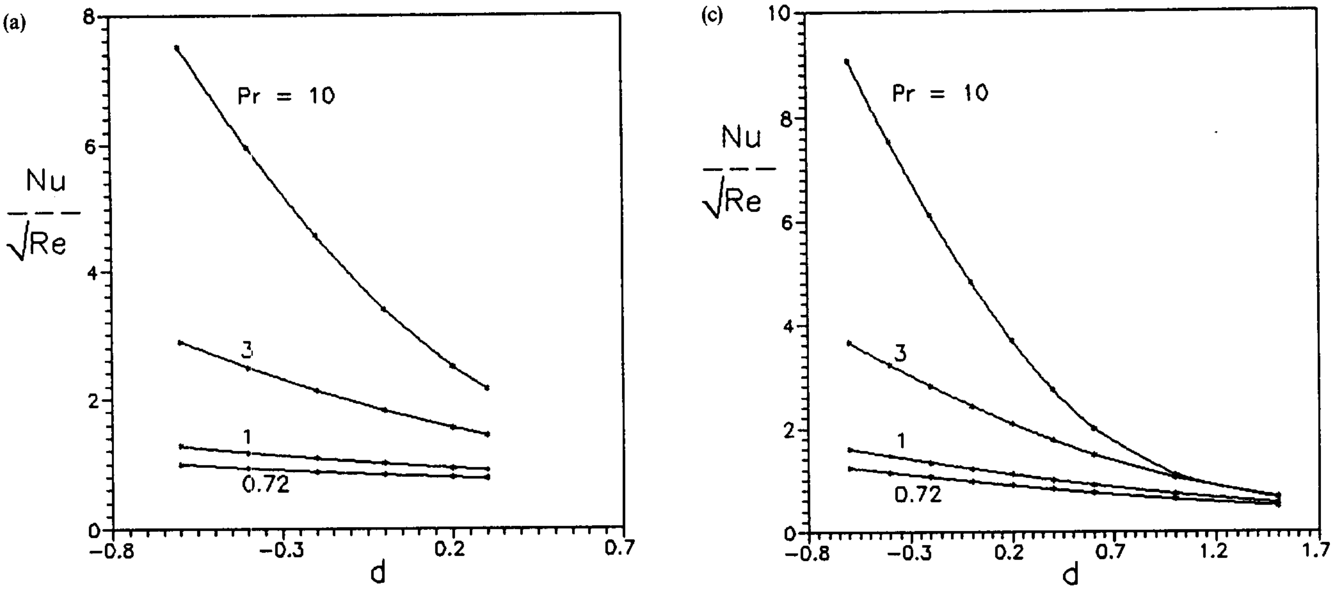
Table 3 $Nu/Re^{1/2}$ for variable surface temperature for various values of Pr as a function of velocity exponent m, injection parameter d, temperature exponent n

m	d	n	Pr = 0.72	Pr = 1.0	Pr = 3.0	Pr = 10.0
−0.2	−0.6	−1.0	0.0713869	0.1520088	1.0491640	5.0221500
−0.2	−0.5	−1.0	−0.0518246	−0.0177133	0.6050459	3.8469160
−0.2	−0.4	−1.0	−0.1912207	−0.2119652	0.0858079	2.5606010
−0.2	−0.3	−1.0	−0.3545958	−0.4464245	−0.5833411	1.0338440
−0.2	−0.2	−1.0	−0.5568877	−0.7518967	−1.6422180	−1.2264180
−0.2	−0.1	−1.0	−0.8280535	−1.2021270	−4.4046570	−9.5806400
−0.2	0.0	−1.0	−1.2382940	−2.0310140	—	—
−0.2	0.1	−1.0	−2.0035990	−4.5387180	—	—
−0.2	0.2	−1.0	−4.2611740	—	—	—

(*continued*)

Table 3 (*continued*)

m	*d*	*n*	Pr = 0.72	Pr = 1.0	Pr = 3.0	Pr = 10.0
−0.2	−0.6	0.0	0.6178836	0.8247324	2.1597620	6.4409420
−0.2	−0.4	0.0	0.5124351	0.6731227	1.6595850	4.6484950
−0.2	−0.2	0.0	0.4175385	0.5355321	1.1986760	2.9672460
−0.2	0.0	0.0	0.3349597	0.4147531	0.7945382	1.5145560
−0.2	0.2	0.0	0.2654861	0.3140982	0.4749947	0.5309956
−0.2	0.3	0.0	0.2355058	0.2708217	0.3495381	0.2513823
−0.2	−0.6	1.0	0.9953792	1.2731300	2.8892090	7.5169230
−0.2	−0.4	1.0	0.9323527	1.1716180	2.4897500	5.9597740
−0.2	−0.2	1.0	0.8794258	1.0834070	2.1327440	4.5640030
−0.2	0.0	1.0	0.8342828	1.0063930	1.8221270	3.3962410
−0.2	0.2	1.0	0.7939048	0.9376202	1.5576520	2.5017430
−0.2	0.3	1.0	0.7746139	0.9054400	1.4415970	2.1565530
1.0	−0.6	−1.0	0.4271666	0.5945808	1.7867090	5.9488090
1.0	−0.4	−1.0	0.2841631	0.3956696	1.1906200	3.9709410
1.0	−0.2	−1.0	0.1410325	0.1969407	0.5929329	1.9848080
1.0	0.2	−1.0	−0.1434538	−0.1978734	−0.5991038	−2.0061600
1.0	0.4	−1.0	−0.2839088	−0.3914412	−1.1769740	−3.8559700
1.0	0.5	−1.0	−0.3529336	−0.4858690	−1.4479860	−4.4055520
1.0	0.6	−1.0	−0.4207611	−0.5780107	−1.6914530	−4.2107370
1.0	0.7	−1.0	−0.4874335	−0.6671232	−1.8868170	−3.2621490
1.0	0.8	−1.0	−0.5522728	−0.7521685	−2.0103170	−2.2675200
1.0	1.0	−1.0	−0.6738980	−0.9023442	−1.9446820	−1.2874600
1.0	1.2	−1.0	−0.7795228	−1.0128580	−1.5436620	−0.9473165
1.0	1.5	−1.0	−0.8880933	−1.0662220	−0.9669135	−0.7121828
1.0	−0.6	0.0	0.7576624	1.0002710	2.4987810	7.0274940
1.0	−0.4	0.0	0.6526364	0.8509860	2.0188300	5.3122310
1.0	−0.2	0.0	0.5535973	0.7103891	1.5690050	3.7106410
1.0	0.0	0.0	0.4614724	0.5799882	1.1597800	2.2958920
1.0	0.2	0.0	0.3773212	0.4615363	0.8025450	1.1739730
1.0	0.4	0.0	0.3015412	0.3563095	0.5098932	0.4466905
1.0	0.6	0.0	0.2350966	0.2659208	0.2906463	0.1118183
1.0	1.0	0.0	0.1311249	0.1312915	0.0633212	0.0016159
1.0	1.5	0.0	0.0523037	0.0420318	0.0042074	0.0000265
1.0	−0.6	1.0	1.0364530	1.3364480	3.0753330	7.9392070
1.0	−0.4	1.0	0.9529272	1.2135290	2.6539630	6.3735120
1.0	−0.2	1.0	0.8758492	1.0999160	2.2646120	4.9436630
1.0	0.0	1.0	0.8055581	0.9959655	1.9143330	3.7004690
1.0	0.2	1.0	0.7417784	0.9018322	1.6072220	2.6984210
1.0	0.4	1.0	0.6679058	0.8076850	1.3425630	1.9593240
1.0	0.6	1.0	0.6322861	0.7420708	1.1314220	1.4674400
1.0	1.0	1.0	0.5436738	0.6167397	0.8242270	0.9404818
1.0	1.5	1.0	0.4558616	0.4991503	0.5990477	0.6446040
5.0	−0.6	−1.0	0.8628425	1.1389490	2.8099510	7.6634000
5.0	−0.4	−1.0	0.7508405	0.9819660	2.3233950	5.9681820
5.0	−0.2	−1.0	0.6419769	0.8294891	1.8560780	4.3583070
5.0	0.2	−1.0	0.4360509	0.5433612	0.9994922	1.5498130
5.0	0.4	−1.0	0.3400072	0.4112284	0.6224660	0.4625075
5.0	0.6	−1.0	0.2492768	0.2876825	0.2903546	−0.3290778
5.0	1.0	−1.0	0.0865183	0.0709186	−0.2111405	−0.9401566
5.0	1.5	−1.0	−0.0762036	−0.1327280	−0.4962238	−0.7301928
5.0	−0.6	0.0	1.0581230	1.3788260	3.2463940	8.3996950
5.0	−0.4	0.0	0.9568765	1.2356160	2.7936280	6.7961990
5.0	−0.2	0.0	0.8592452	1.0978480	2.3626020	5.2913060
5.0	0.0	0.0	0.7659087	0.9641629	1.9523930	3.8926200
5.0	0.2	0.0	0.6772155	0.8424000	1.5833110	2.7124140
5.0	0.4	0.0	0.5934594	0.7258015	1.2438630	1.7201330
5.0	0.6	0.0	0.5149341	0.6174377	0.9451767	0.9722751
5.0	1.0	0.0	0.3753285	0.4284874	0.4824506	0.1970245
5.0	1.5	0.0	0.2356861	0.2472617	0.1564606	0.0103238
5.0	−0.6	1.0	1.2405660	1.6015050	3.6459380	9.0780400
5.0	−0.4	1.0	1.1479910	1.4692510	3.2190180	7.5458460
5.0	−0.2	1.0	1.0592520	1.3426930	2.8139110	6.1171200
5.0	0.0	1.0	0.9749730	1.2225480	2.4354270	4.8193010
5.0	0.2	1.0	0.8952823	1.1090340	2.0859960	3.6834960
5.0	0.4	1.0	0.8199229	1.0031480	1.7691640	2.7373290
5.0	0.6	1.0	0.7498769	0.9047923	1.4880880	1.9979590
5.0	1.0	1.0	0.6249081	0.7320322	1.0391360	1.1019880
5.0	1.5	1.0	0.4977645	0.5616040	0.6787248	0.6665520



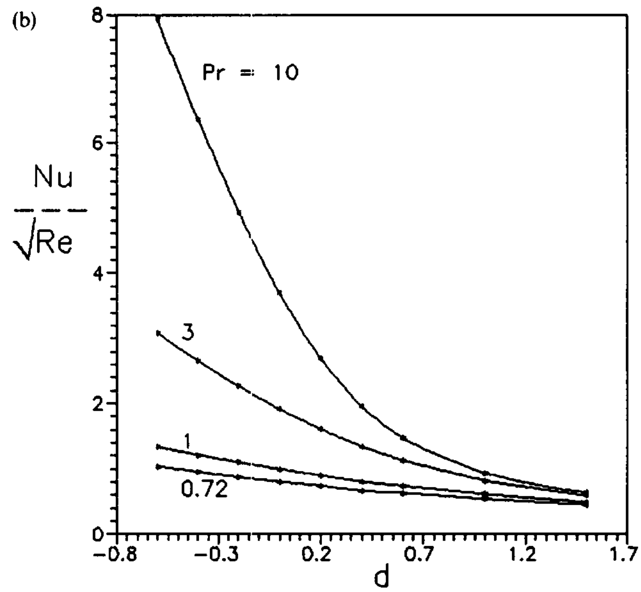


Figure 5 Variation of $Nu/Re^{1/2}$ as a function of the injection parameter d for variable surface temperature, $n = 1$, and for various values of Prandtl number; (a) $m = -0.2$; (b) $m = 1$; (c) $m = 5$

of d, and for different combinations of n and m according to the relation $n = 0.5(1 - m)$. Figure 6a is for $n = -1$, $m = 3$; Figure 6b is for $n = 0$, $m = 1$; and Figure 6c is for $n = 0.6$ and $m = -0.2$, respectively, and it is clear that suction decreases the thermal boundary layer; whereas, blowing increases it, and in all cases, heat is transferred from the moving surface to the medium.

Comparing Figure 3b to that of Figure 6b, both are for $Pr = 0.72$, $n = 0$, and $m = 1$, it can be seen that, for the range of blowing parameter used, the thermal boundary layer is reduced slightly when the surface heat flux is uniform.

The dimensionless heat transfer coefficient $Nu/Re^{1/2}$ for uniform surface heat flux is presented for various values of Pr of 0.72, 1, 3, and 10 and for $n = -1$, 0, 0.6, which gives m of 3, 1, and -0.2, respectively, in Figure 7a–c. In Figure 7a, the local $Nu/Re^{1/2}$ is found to vary with the blowing parameter d, and a linear variation is a reasonable approximation for small Pr, and for $Pr = 10$, it has a slight curvature.

The dimensionless temperature $\Theta(0)$ increases with increasing d from suction to blowing for $n = -1$ up to a certain values of d and Pr, where $\Theta(0)$ decreases as d increases. Furthermore, beyond these, certain values of $\Theta(0)$ are negative, which indicates that $\Theta(\eta)$ has a region of temperature less than ambient temperature. Despite the fact that these solutions satisfy the governing equations and the boundary conditions, they are physically unrealistic, and the corresponding values of d and Pr are not reported in Figure 7a, and they are presented as dashed lines in Table 4.

Figures 7b and c show the $Nu/Re^{1/2}$ variation with the blowing parameter d for $n = 0$ and 0.6, respectively, and it

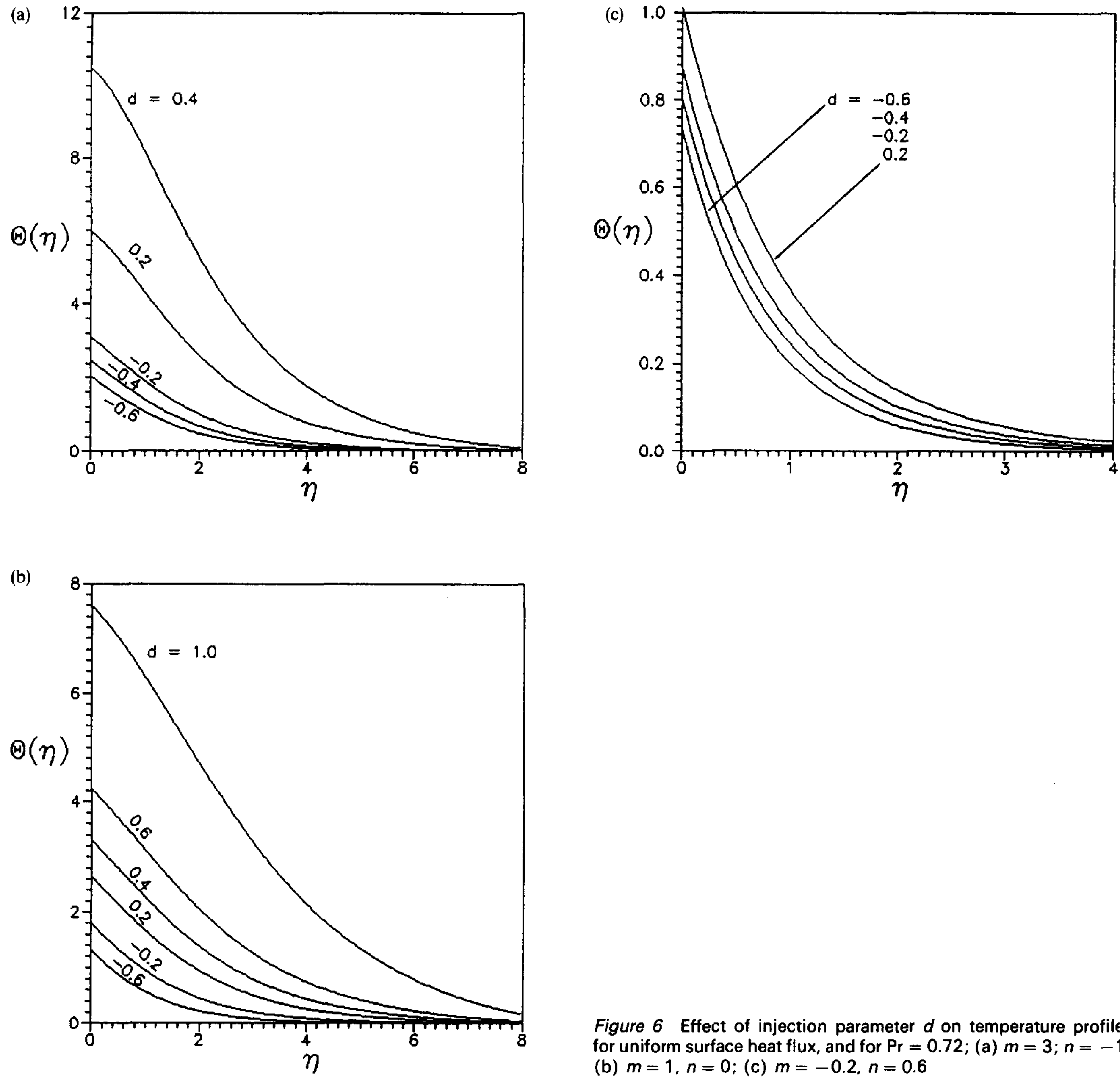


Figure 6 Effect of injection parameter d on temperature profiles for uniform surface heat flux, and for $Pr = 0.72$; (a) $m = 3$; $n = -1$; (b) $m = 1$, $n = 0$; (c) $m = -0.2$, $n = 0.6$

should be noted that the values of $Nu/Re^{1/2}$ beyond $d = 0.2$ were discarded, because the corresponding dimensionless velocity profiles $f'(\eta)$ for $m = -0.2$ have a reverse flow, as given by Ali (1996).

Summary and conclusions

Heat transfer characteristics of a continuously stretched surface with suction or injection are discussed for three boundary conditions. A similarity transformation was used to solve the laminar momentum and energy boundary-layer equations.

It is shown that suction increases the heat transfer from the surface; whereas, injection causes a decrease in the heat transfer for all parameters studied. For negative values of n, heat flows to or from the stretched surface depending upon m, d, and Pr; however, for $n = -1$ and $m = 1$ and for all Prandtl numbers investigated, there is no heat exchange between the surface and the ambient at $d = 0$. Furthermore, for positive values of n, all m, and Pr, it was found that for all suction or injection parameters studied, the heat is transferred from the surface to the medium.

It was found that the dimensionless local heat transfer coefficient increases with decreasing injection parameter d, and with increasing the velocity and temperature exponents m and n, respectively, for constant Prandtl number. Finally, it was shown that increasing Prandtl number enhances the heat transfer coefficient, keeping all other parameters constant.

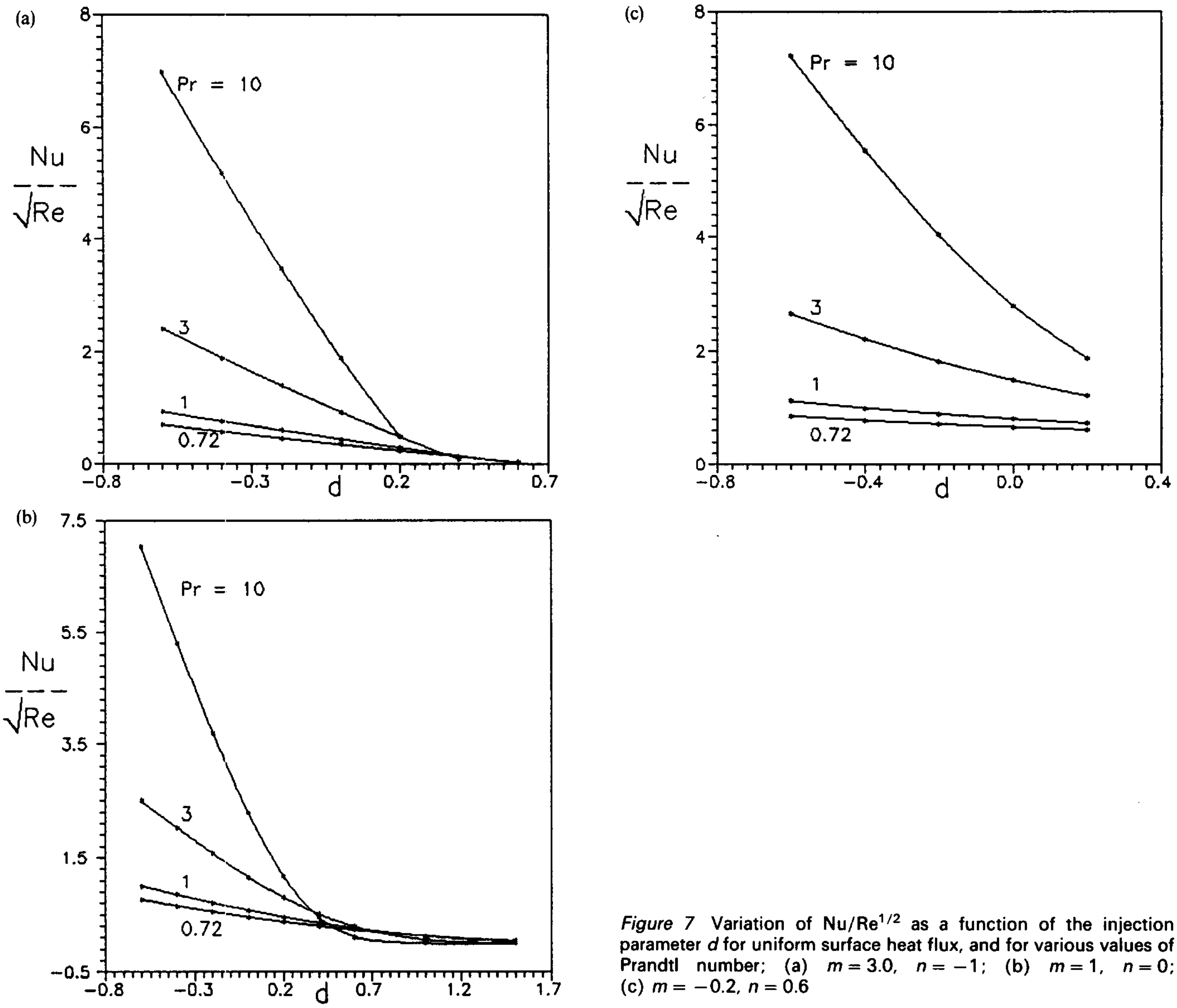


Figure 7 Variation of $Nu/Re^{1/2}$ as a function of the injection parameter d for uniform surface heat flux, and for various values of Prandtl number; (a) $m = 3.0$, $n = -1$; (b) $m = 1$, $n = 0$; (c) $m = -0.2$, $n = 0.6$

Table 4 $Nu/Re^{1/2}$ for uniform surface heat flux for various values of Pr as a function of velocity exponent m, injection parameter d, and temperature exponent n

m	d	n	Pr = 0.72	Pr = 1.0	Pr = 3.0	Pr = 10.0
3.0	−0.6	−1.0	0.69483	0.92798	2.40737	6.97896
3.0	−0.4	−1.0	0.57458	0.75985	1.88909	5.18102
3.0	−0.2	−1.0	0.45773	0.59708	1.39071	3.46426
3.0	0.0	−1.0	0.34507	0.44036	0.91765	1.87528
3.0	0.2	−1.0	0.23703	0.29128	0.47952	0.48858
3.0	0.4	−1.0	0.13489	0.15098	0.08802	—
3.0	0.6	−1.0	0.03904	0.02180	—	—
1.0	−0.6	0.0	0.76235	1.00630	2.51599	7.09238
1.0	−0.4	0.0	0.65637	0.85570	2.03115	5.35248
1.0	−0.2	0.0	0.55653	0.71398	1.57752	3.73345
1.0	0.2	0.0	0.37916	0.46373	0.80621	1.17981
1.0	0.4	0.0	0.30304	0.35803	0.51233	0.44921
1.0	0.6	0.0	0.23632	0.26722	0.29215	0.11247
1.0	1.0	0.0	0.13182	0.13205	0.06363	0.00175
1.0	1.5	0.0	0.05267	0.04222	0.00423	0.00011
−0.2	−0.6	0.6	0.86356	1.11842	2.64873	7.21160
−0.2	−0.4	0.6	0.78788	1.00190	2.21411	5.54308
−0.2	−0.2	0.6	0.72419	0.90095	1.82667	4.04361
−0.2	0.0	0.6	0.67110	0.81476	1.49406	2.80052
−0.2	0.2	0.6	0.62599	0.74053	1.21911	1.88849

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